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II. Solution by P. H. PHILBRICK, C. E., Lake Charles, Louisiana.

The above expression as we will see is not correct.

We have from Trigonometry $\sin x = x\left(1-\frac{x^2}{1.2.3}+\frac{x^4}{1.2.3.4.5}-\text{etc.}\right)$.

To resolve the terms in the parenthesis into factors, we observe that they must be of the form $1-\frac{x^2}{q}$, since the coefficint of x^0 is 1, and the exponents differ by 2.

Moreover, they must be such as to reduce the second member to zero for $x=\pm n\pi$, or $x^2=n^2\pi^2$, for such values of x reduce the first member to zero. Hence, $q=n^2\pi^2$. Making n successively 1, 2, 3, etc. the above becomes:

$$\sin x = x\left(1-\frac{x^2}{1^2\pi^2}\right)\left(1-\frac{x^2}{2^2\pi^2}\right)\left(1-\frac{x^2}{3^2\pi^2}\right)\ldots.$$

Putting $x=\frac{\pi}{2}$ we have,

$$\sin\frac{\pi}{2}=1=\frac{\pi}{2}\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{4^2}\right)\left(1-\frac{1}{6^2}\right)\ldots.$$

$$=\frac{\pi}{2}\left(\frac{2^2-1}{2^2}\right)\left(\frac{4^2-1}{4^2}\right)\left(\frac{6^2-1}{6^2}\right)\ldots.$$

$$=\frac{\pi}{2}\cdot\frac{2-1}{2}\cdot\frac{2+1}{2}\cdot\frac{4-1}{4}\cdot\frac{4+1}{4}\cdot\frac{6-1}{6}\cdot\frac{6+1}{6}\ldots.$$

$$\text{Therefore}\ \frac{\pi}{2}=\left(\frac{2}{1}\cdot\frac{2}{3}\right)\left(\frac{4}{3}\cdot\frac{4}{5}\right)\left(\frac{6}{5}\cdot\frac{6}{7}\right)\ldots.$$

$$=\left[\frac{2.\ 4.\ 6.\ 8.\ldots.2n}{1.\ 3.\ 5.\ 7.\ldots.2n-1}\right]^2\frac{1}{2n+1},\ \text{or}$$

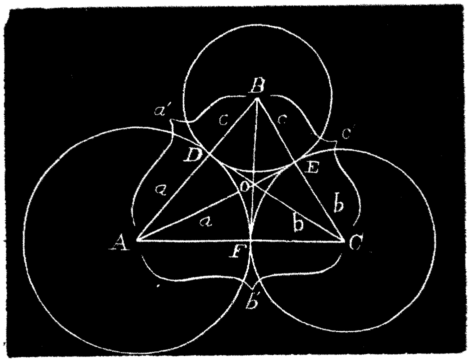
$$=\left[\frac{2.\ 4.\ 6.\ 8.\ldots.(2n)}{1.\ 3.\ 5.\ 7.\ldots.(2n+1)}\right]^2\frac{2n+1}{1}.$$

4. Three circles whose radii are a, b, and c touch each other externally; prove that the tangents at the points of contact meet in a point whose distance from any one of them is $\left[\frac{abc}{a+b+c}\right]^{\frac{1}{2}}$. [Selected from *Todhunter's Plane Trigonometry.*]

I. Solution by Professor G. B. M. ZERR, A. M., Principal of Schools, Staunton, Virginia.

Let $OD=OE=OF=r$. Then $\frac{r}{2}(a'+b'+c')=r(a+b+c)=\triangle$ where $\triangle$ = area of triangle; but $\triangle=\sqrt{\tfrac{1}{2}(a'+b'+c')\tfrac{1}{2}(a'+b'-c')\tfrac{1}{2}(a'+c'-b')\tfrac{1}{2}(b'+c'-a')}=\sqrt{(a+b+c)abc}$. $\therefore\ r(a+b+c)=\sqrt{(a+b+c)\,abc}$.

$$\therefore\ r=\left(\frac{abc}{a+b+c}\right)^{\frac{1}{2}}.$$



This problem was solved in a very excellent manner by *P. S. Berg, H. C. Whitaker*, and *P. H. Philbrick*.

II. Solution by Professor J. F. W. SCHEFFER, A. M., Hagerstown, Maryland.

Denote the circles whose radii are a, b, and c respectively by O, O', O'', and P the point of intersection of the tangents common to O, O' and to O', O''. Since P is in the tangent common to O and O', the powers of P with reference to these two circles are equal, and as much as P is in the common tangent of O' and O'', its powers to these two circles are equal; consequently, its powers with ref. to O and O'' being equal, P must lie in the tangent common to O and O''. It is, in fact, the radical center of the three radical axes. Denoting the required distance by x, we have for the areas of the three triangles $OO'P$, $OO''P$, $O'O''P$, respectively, $\frac{1}{2}x(a+b)$, $\frac{1}{2}x(a+c)$, $\frac{1}{2}x(b+c)$; and since the area of $\triangle OO'O''$, whose sides are $a+b$, $a+c$, $b+c$, is $=\sqrt{abc(a+b+c)}$, we have $\frac{1}{2}x(a+b)+\frac{1}{2}x(a+c)+\frac{1}{2}x(b+c)=\sqrt{abc(a+b+c)}$, whence $x=\frac{\sqrt{abc(a+b+c)}}{a+b+c}=\sqrt{\frac{abc}{a+b+c}}$.

Q. E. D.

5. Proposed by ADOLPH BAILOFF, Durand, Wisconsin.

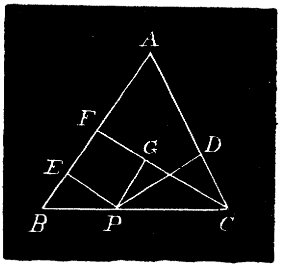
If from a variable point in the base of an isosceles triangle, perpendiculars are drawn to the sides, the sum of the perpendiculars is constant and equal to the perpendicular let fall from either extremity of the base to the opposite side.

I. Solution by ROBERT J. ALEY, A. M., Professor of Mathematics in the Indiana State University, Bloomington, Indiana.

PG is drawn paralell to AB.

$PE=FG$.

$PD=GC$, because $\triangle\ GPC$ and $\triangle\ PDC$ have the same hypotenuse PC and the acute angle GPC of the one equal to the acute angle DCP of the other. $\therefore\ PE+PD=FG+GC=FC$.

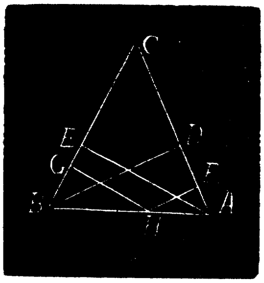


II. Solution by Hon. JOSIAH H. DRUMMOND, LL. D., Portland, Maine;

Connecting the vertex of the triangle with the variable point, wherever in the base it may be, the triangle is divided into two triangles whose area is one-half the product of the perpendicular and one of the equal sides of the triangle respectively; hence, the area of the original triangle is equal to one-half the product of one the equal sides by the sum of the two perpendiculars. Hence the sum of the perpendiculars equals the perpendicular from the extremity of the base to the opposite side.

Solved similarly by *P. H. Philbrick*

III. Solution by A. L. FOOTE, C. E., No. 80, Broad St., New York, and ALBERT J. ROBINSON, Pottersburg, Ohio.



The triangle being isosceles, the angles CAB CBA and are equal, consequently BD and AE are equal, and because they are at right angles to the sides AC and BC, then HF and HG which are at right angles to AC and BC are parallel, HF to BD and HG to AE; whence, $BH:AB::GH:AE$ and $AH:AB::FH:BD$. Therefore, $AH+BH:2AB::GH+FH:AE+BD$. But $BD=AE$ and $AH+BH=AB$, then we have $AB:2AB::GH+FH:2AE$ or